

$B \rightarrow 0^+ (1^+) + \text{missing energy in Unparticle Physics}$

M. Jamil Aslam^{1,2} and Cai-Dian Lü¹

¹*Institute of High Energy Physics, P.O. Box 918(4), Beijing 100049, China and*

²*Department of Physics, COMSATS Institute of Information Technology, Islamabad, Pakistan*

We examine the effects of an unparticle \mathcal{U} as a possible source of missing energy in the p -wave decays of B meson. The dependence of the differential branching ratio on the $K_0^* (K_1)$ -meson's energy is discussed in the presence of scalar and vector unparticle operators and significant deviation from the standard model value is found after addition of these operators. Finally, we have shown the dependence of branching ratio for the above said decays on the parameters of the unparticle stuff like the effective couplings, cutoff scale $\Lambda_{\mathcal{U}}$ and the scale dimensions $d_{\mathcal{U}}$.

I. INTRODUCTION

Flavor changing neutral current (FCNC) processes induced by $b \rightarrow s$ transitions are not allowed at tree level in the Standard Model (SM), but are generated at loop level and are further suppressed by the CKM factors. Therefore, these decays are very sensitive to the physics beyond the SM via the influence of new particles in the loop. Though the branching ratios of FCNC decays are small in the SM, quite interesting results are obtained from the experiments both for the inclusive $B \rightarrow X_s \ell^+ \ell^-$ [1] and exclusive decay modes $B \rightarrow K \ell^+ \ell^-$ [2, 3, 4] and $B \rightarrow K^* \ell^+ \ell^-$ [5]. These results are in good agreement with the theoretical estimates [6, 7, 8].

Among different semileptonic decays induced by $b \rightarrow s$ transitions, $b \rightarrow s \nu \bar{\nu}$ decays are of particular interest, because of the absence of a photonic penguin contribution and hadronic long distance effects these have much smaller theoretical uncertainties. But experimentally, it is too difficult to measure the inclusive decay modes $B \rightarrow X_s \nu \bar{\nu}$ as one has to sum on all the X_s 's. Therefore, the exclusive $B \rightarrow K (K^*) \nu \bar{\nu}$ decays play a peculiar role both from the experimental and theoretical point of view. The theoretical estimates of the branching ratio of these decays are $Br(B \rightarrow K \nu \bar{\nu}) \sim 10^{-5}$ and $Br(B \rightarrow K^* \nu \bar{\nu}) \sim 10^{-6}$ [9] whereas, the experimental bounds given by the B -factories, BELLE and BaBar, on these decays are [10, 11]:

$$Br(B \rightarrow K \nu \bar{\nu}) < 1.4 \times 10^{-5} \quad (1)$$

$$Br(B \rightarrow K^* \nu \bar{\nu}) < 1.4 \times 10^{-4}.$$

These processes, based on $b \rightarrow s \nu \bar{\nu}$, are very sensitive to the new physics and have been studied

extensively in the literature in the context of large extra dimension model and Z' models [12, 13]. Any new physics model which can provide a relatively light new source of missing energy (which is attributed to the neutrinos in the SM) can potentially enhance the observed rates of $B \rightarrow K(K^*) + \text{missing energy}$. Recently, H. Georgi has proposed one such model of Unparticles, which is one of the tantalizing issues these days [14]. The main idea of Georgi's model is that at a very high energy our theory contains the fields of the standard model and the fields of a theory with a nontrivial IR fixed point, which he called BZ (Banks-Zaks) fields [15]. The interaction among the two sets is through the exchange of particles with a large mass scale M_U . The coupling between the SM fields and BZ fields are nonrenormalizable below this scale and are suppressed by the powers of M_U . The renormalizable couplings of the BZ fields then produce dimensional transmutation and the scale invariant unparticle emerged below an energy scale Λ_U . In the effective theory below the scale Λ_U the BZ operators matched onto unparticle operators, and the renormalizable interaction matched onto a new set of interactions between standard model and unparticle fields. The outcome of this model is the collection of unparticle stuff with scale dimension d_U , which is just like a non-integral number of invisible massless particles, whose production might be detectable in missing energy and momentum distributions [16].

This idea promoted a lot of interest in unparticle physics and its signatures have been discussed at colliders [16, 17, 18, 19, 20], in low energy physics [21], Lepton Flavor Violation [22], unparticle physics effects in B_s mixing [23], and also in cosmology and astrophysics [24]. Aliev et al. have studied $B \rightarrow K(K^*) + \text{missing energy}$ in unparticle physics [25] in which they have studied the effects of an unparticle U as a possible source of a missing energy in these decays. They have found the dependence of the differential branching ratio on the $K(K^*)$ -meson's energy in the presence of scalar and vector unparticle operators and then using the upper bounds on these decays, they put stringent constraints on the parameters of the unparticle stuff.

The studies are even more complete if similar studies for the p-wave decays of B meson such as $B \rightarrow K_0^*(1430) + \cancel{E}$ (\cancel{E} is missing energy) and $B \rightarrow K_1(1270) + \cancel{E}$, where $K_0^*(1430)$ and $K_1(1270)$ are the pseudoscalar and axial vector mesons respectively, carried out. In this paper, we have studied these p-wave decays of B meson in unparticle physics using the frame work of Aliev et al. [25]. We have considered the decay $B \rightarrow K_0^*(K_1)\nu\bar{\nu}$ in SM although for these modes no signals have been observed so far, but in future B-factories where enough data is expected, these decays will be observed. These SuperB factories will be measuring these processes by analyzing the spectra of the final state hadron. In doing this measurement a cut for high momentum on the hadron is imposed, in order to suppress the background. Therefore, unparticle would give us a

unique distribution for the high energy hadron in the final state, such that in future B-factories one will be able to distinguish the presence of unparticle by observing the spectrum of final state hadrons in $B \rightarrow (K, K^*, K_0^*, K_1) + \cancel{E}$ [25].

The work is organized as follows: In section II after giving the expression for the effective Hamiltonian for the decay $b \rightarrow s\nu\bar{\nu}$, we define the scalar and vector unparticle physics operators for $b \rightarrow s\mathcal{U}$. Then using these expressions we calculate the various contributions the decay rates of $B \rightarrow K_0^*(K_1) + \cancel{E}$ both from the SM and unparticle theory in Section III. Recently, Grinstein et al. gave comments on the unparticle [26] mentioning that Mack's unitarity constraint lower bounds on CFT operator dimensions, e.g $d_{\mathcal{U}} \geq 3$ for primary, gauge invariant, vector unparticle operators. To account for this they have corrected the results in the literature, and modified the propagator of vector and tensor unparticles. We will also give the expressions of decay rate using these modified vector operators in the same section. Finally, section IV contains our numerical results and conclusions.

II. EFFECTIVE HAMILTONIAN IN SM AND UNPARTICLE OPERATORS

The flavor changing neutral current $b \rightarrow s\nu\bar{\nu}$ are of particular interest both from theoretical and experimental view. One of the main reason of interest is the absence of long distance contribution related to the four-quark operators in the effective Hamiltonian. In this respect, the transition to neutrino represents a clean process even in comparison with the $b \rightarrow s\gamma$ decay, where long-distance contributions, though small, are expected to present [27]. In Standard Model these processes are governed by the effective Hamiltonian

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi} V_{tb} V_{ts}^* C_{10} \bar{s} \gamma^\mu (1 - \gamma_5) b \bar{\nu} \gamma_\mu (1 - \gamma_5) \nu \quad (2)$$

where $V_{tb} V_{ts}^*$ are the elements of the Cabbibo-Kobayashi Maskawa Matrix and C_{10} is obtained from the Z^0 penguin and box diagrams where the dominant contribution corresponds to a top quark intermediate state and it is

$$C_{10} = \frac{D(x_t)}{\sin^2 \theta_w}. \quad (3)$$

θ_w is the Weinberg angle and $D(x_t)$ is the usual Inami-Lim function, given as

$$D(x_t) = \frac{x_t}{8} \left\{ \frac{x_t + 2}{x_t - 1} + \frac{3x_t - 6}{(x_t - 1)^2} \ln(x_t) \right\}, \quad (4)$$

with $x_t = m_t^2/m_W^2$.

The unparticle transition at the quark level can be described by $b \rightarrow s\mathcal{U}$, where one can consider the following operators:

- Scalar unparticle operator

$$C_s \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}} \bar{s} \gamma_{\mu} b \partial^{\mu} O_{\mathcal{U}} + C_P \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}} \bar{s} \gamma_{\mu} \gamma_5 b \partial^{\mu} O_{\mathcal{U}} \quad (5)$$

- Vector unparticle operator

$$C_V \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}} \bar{s} \gamma_{\mu} b O_{\mathcal{U}}^{\mu} + C_A \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}} \bar{s} \gamma_{\mu} \gamma_5 b O_{\mathcal{U}}^{\mu}. \quad (6)$$

The propagator for the scalar unparticle field can be written as [14, 16, 17]

$$\int d^4x e^{iP \cdot x} \langle 0 | T O_{\mathcal{U}}(x) O_{\mathcal{U}}(0) | 0 \rangle = i \frac{A_{d_{\mathcal{U}}}}{2 \sin(d_{\mathcal{U}} \pi)} (-P^2)^{d_{\mathcal{U}}-2} \quad (7)$$

with

$$A_{d_{\mathcal{U}}} = \frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}} \frac{\Gamma(d_{\mathcal{U}} + 1/2)}{\Gamma(d_{\mathcal{U}} - 1) \Gamma(2d_{\mathcal{U}})}. \quad (8)$$

III. DIFFERENTIAL DECAY WIDTHS

In Standard Model the decay $B \rightarrow K_0^*(K_1) + \ell \bar{\nu}$ is described by the decay $B \rightarrow K_0^*(K_1) \nu \bar{\nu}$. At quark level this process is governed by the effective Hamiltonian defined in Eq. (2) which when sandwiched between B and $K_0^*(K_1)$ involves the hadronic matrix elements for the exclusive decay $B \rightarrow K_0^*(K_1) \nu \bar{\nu}$. These can be parameterized by the form factors and the non-vanishing matrix elements for $B \rightarrow K_0^*$ are [27]

$$\langle K_0^*(p') | \bar{s} \gamma_{\mu} \gamma_5 b | B(p) \rangle = -i \left[f_+(q^2) (p + p')_{\mu} + f_-(q^2) q_{\mu} \right]. \quad (9)$$

where $q_{\mu} = (p + p')_{\mu}$. Using the above definition and taking into account the three species of neutrinos in the Standard Model, the differential decay width as a function of K_0^* energy ($E_{K_0^*}$) can be written as [27]:

$$\frac{d\Gamma^{SM}}{dE_{K_0^*}} = \frac{G_F^2 \alpha^2}{2^7 \pi^5 M_B^2} |V_{tb} V_{ts}^*|^2 |C_{10}|^2 f_+^2(q^2) \sqrt{\lambda^3(M_B^2, M_{K_0^*}^2, q^2)} \quad (10)$$

with $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca$ and $q^2 = M_B^2 + M_{K_0^*}^2 - 2M_B E_{K_0^*}$. Here $f_+(q^2)$ and $f_-(q^2)$ are the form factors which are the non-perturbative quantities and can be calculated

using some models. The one we have used here was calculated by using Light Front Quark Model (LFQR) by Cheng et al. [27] and these can be parameterized as:

$$F(q^2) = \frac{F(0)}{1 - aq^2/M_B^2 + b(q^2/M_B^2)^2}$$

and the fitted parameters are given in Table I.

TABLE I: The parameters for $B \rightarrow K_0^*$ form factors.

	$F(0)$	a	b
f_+	-0.26	1.36	0.86
f_-	0.21	1.26	0.93

Similarly, for $B \rightarrow K_1$ transition the matrix elements can be parametrized as [28]

$$\begin{aligned} \langle K_1(k, \varepsilon) | V_\mu | B(p) \rangle &= i\varepsilon_\mu^* (M_B + M_{K_1}) V_1(q^2) \\ &\quad - (p+k)_\mu (\varepsilon^* \cdot q) \frac{V_2(q^2)}{M_B + M_{K_1}} \\ &\quad - q_\mu (\varepsilon \cdot q) \frac{2M_{K_1}}{s} [V_3(q^2) - V_0(q^2)] \end{aligned} \quad (11)$$

$$\langle K_1(k, \varepsilon) | A_\mu | B(p) \rangle = \frac{2i\epsilon_{\mu\nu\alpha\beta}}{M_B + M_{K_1}} \varepsilon^{*\nu} p^\alpha k^\beta A(q^2) \quad (12)$$

where $V_\mu = \bar{s}\gamma_\mu b$ and $A_\mu = \bar{s}\gamma_\mu\gamma_5 b$ are the vector and axial vector currents respectively and ε_μ^* is the polarization vector for the final state axial vector meson. In this case we have used the form factors that were calculated by Paracha et al. [28] and the corresponding expressions are:

$$\begin{aligned} A(s) &= \frac{A(0)}{(1 - s/M_B^2)(1 - s/M_B'^2)} \\ V_1(s) &= \frac{V_1(0)}{(1 - s/M_{B_A}^2)(1 - s/M_{B_A}'^2)} \left(1 - \frac{s}{M_B^2 - M_{K_1}^2} \right) \\ V_2(s) &= \frac{\tilde{V}_2(0)}{(1 - s/M_{B_A}^2)(1 - s/M_{B_A}'^2)} - \frac{2M_{K_1}}{M_B - M_{K_1}} \frac{V_0(0)}{(1 - s/M_B^2)(1 - s/M_B'^2)} \end{aligned} \quad (13)$$

with

$$\begin{aligned} A(0) &= -(0.52 \pm 0.05) \\ V_1(0) &= -(0.24 \pm 0.02) \\ \tilde{V}_2(0) &= -(0.39 \pm 0.03). \end{aligned} \quad (14)$$

The differential decay rate can be calculated as [25]:

$$\frac{d\Gamma^{SM}}{dE_{K_1}} = \frac{G_F^2 \alpha^2}{2^9 \pi^5 M_B^2} |V_{tb} V_{ts}^*|^2 |C_{10}|^2 \lambda^{1/2} |M_{SM}|^2 \quad (15)$$

where

$$|M_{SM}|^2 = \frac{8q^2 \lambda |A(q^2)|^2}{(M_B + M_{K_1})^2} + \frac{1}{M_{K_1}^2} \left[\lambda^2 \frac{|V_2(q^2)|^2}{(M_B + M_{K_1})^2} + (M_B + M_{K_1})^2 (\lambda + 12M_{K_1}^2 q^2) |V_1(q^2)|^2 \right. \\ \left. - \lambda (M_B^2 - M_{K_1}^2 - q^2) \operatorname{Re}(V_1^*(q^2) V_2(q^2) + V_2^*(q^2) V_1(q^2)) \right] \quad (16)$$

and $\lambda = \lambda(M_B^2, M_{K_1}^2, q^2)$ with $q^2 = M_B^2 + M_{K_1}^2 - 2M_B E_{K_1}$.

Now in decay mode $B \rightarrow K_0^*(K_1) + \cancel{E}$, the missing energy shown by \cancel{E} can also be attributed to the unparticle and hence the unparticle can also contribute to these decay modes. Therefore, the signature of two decay modes $B \rightarrow K_0^*(K_1) \nu \bar{\nu}$ and $B \rightarrow K_0^*(K_1) \mathcal{U}$ is required like the one done for $B \rightarrow K(K^*) \nu \bar{\nu}$ and $B \rightarrow K(K^*) \mathcal{U}$ in the literature [25].

A. The Scalar Unparticle Operator

Using the scalar unparticle operator defined in Eq. (5) the matrix element for $B \rightarrow K_0^* \mathcal{U}$ can be written as

$$\mathcal{M}_{K_0^*}^{SU} = \frac{1}{\Lambda^{d_{\mathcal{U}}}} \langle K_0^*(p') | \bar{s} \gamma_\mu (\mathcal{C}_S + \mathcal{C}_P \gamma_5) b | B(p) \rangle \partial^\mu O_{\mathcal{U}} \\ = \frac{1}{\Lambda^{d_{\mathcal{U}}}} \mathcal{C}_P [f_+(q^2) (M_B^2 - M_{K_0^*}^2) + f_-(q^2) q^2] O_{\mathcal{U}} \quad (17)$$

Now the decay rate for $B \rightarrow K_0^* \mathcal{U}$ can be evaluated to be:

$$\frac{d\Gamma^{SU}}{dE_{K_0^*}} = \frac{1}{8\pi^2 m_B} \sqrt{E_{K_0^*}^2 - M_{K_0^*}^2} |\mathcal{M}^{SU}|^2 \quad (18)$$

where

$$|\mathcal{M}^{SU}|^2 = |\mathcal{C}_P|^2 \frac{A_{d_{\mathcal{U}}}}{\Lambda^{2d_{\mathcal{U}}}} \left(M_B^2 + M_{K_0^*}^2 - 2M_B E_{K_0^*} \right)^{d_{\mathcal{U}}-2} \\ \times \left[f_+(q^2) (M_B^2 - M_{K_0^*}^2) + f_-(q^2) (M_B^2 + M_{K_0^*}^2 - 2M_B E_{K_0^*}) \right]^2. \quad (19)$$

Following the same lines, the corresponding matrix element $B \rightarrow K_1 \mathcal{U}$ is

$$\mathcal{M}_{K_1}^{SU} = \frac{1}{\Lambda^{d_{\mathcal{U}}}} \langle K_1(p') | \bar{s} \gamma_\mu (\mathcal{C}_S + \mathcal{C}_P \gamma_5) b | B(p) \rangle \partial^\mu O_{\mathcal{U}} \\ = \frac{i}{\Lambda^{d_{\mathcal{U}}}} \mathcal{C}_S (\varepsilon^* \cdot q) \left[(M_B + M_{K_1}) V_1(q^2) \right. \\ \left. - (M_B - M_{K_1}) V_2(q^2) - 2M_{K_1} (V_3(q^2) - V_0(q^2)) \right] O_{\mathcal{U}}, \quad (20)$$

and the differential decay rate is

$$\frac{d\Gamma^{SU}}{dE_{K_1}} = \frac{M_B}{2\pi^2} \frac{A_{d_{\mathcal{U}}}}{\Lambda^{2d_{\mathcal{U}}}} |\mathcal{C}_S|^2 |V_0(q^2)|^2 (E_{K_1}^2 - M_{K_1}^2)^{3/2} (M_B^2 + M_{K_1}^2 - 2M_B E_{K_1})^{d_{\mathcal{U}}-2}. \quad (21)$$

One can see from Eq. (18) and Eq. (21) that the scalar unparticle contribution to the decay rate depends on \mathcal{C}_P , \mathcal{C}_S , $d_{\mathcal{U}}$ and $\Lambda_{\mathcal{U}}$, therefore one can see the behavior of decay rates for the said decays on these parameters which will be hoped to get constraint once we have experimental data on these decays. This we will do in a separate section.

B. The Vector Unparticle Operator

The matrix element for $B \rightarrow K_0^* \mathcal{U}$ using the vector unparticle operator defined in Eq. (6) and the definition of form factors given in Eq. (9) can be calculated as:

$$\begin{aligned} \mathcal{M}_{K_0^*}^{V\mathcal{U}} &= \frac{1}{\Lambda^{d_{\mathcal{U}}-1}} \langle K_0^* (p') | \bar{s} \gamma_{\mu} (\mathcal{C}_V + \mathcal{C}_A \gamma_5) b | B (p) \rangle O_{\mathcal{U}}^{\mu} \\ &= \frac{1}{\Lambda^{d_{\mathcal{U}}-1}} \mathcal{C}_A [f_+ (q^2) (p + p')_{\mu} + f_- (q^2) q_{\mu}] O_{\mathcal{U}}^{\mu}. \end{aligned} \quad (22)$$

The differential decay rate is then

$$\begin{aligned} \frac{d\Gamma^{V\mathcal{U}}}{dE_{K_0^*}} &= \frac{1}{8\pi^2 m_B} \frac{A_{d_{\mathcal{U}}}}{\Lambda^{2d_{\mathcal{U}}-2}} |\mathcal{C}_A|^2 |f_+ (q^2)|^2 \left(M_B^2 + M_{K_0^*}^2 - 2M_B E_{K_0^*} \right)^{d_{\mathcal{U}}-2} \sqrt{E_{K_0^*}^2 - M_{K_0^*}^2} \\ &\times \left\{ - \left(M_B^2 + M_{K_0^*}^2 + 2M_B E_{K_0^*} \right) + \frac{\left(M_B^2 - M_{K_0^*}^2 \right)^2}{\left(M_B^2 + M_{K_0^*}^2 - 2M_B E_{K_0^*} \right)} \right\}. \end{aligned} \quad (23)$$

For $B \rightarrow K_1$ case the matrix element for $B \rightarrow K_1 \mathcal{U}$ is

$$\begin{aligned} \mathcal{M}_{K_1}^{V\mathcal{U}} &= \frac{1}{\Lambda^{d_{\mathcal{U}}-1}} \langle K_1 (p') | \bar{s} \gamma_{\mu} (\mathcal{C}_V + \mathcal{C}_A \gamma_5) b | B (p) \rangle O_{\mathcal{U}}^{\mu} \\ &= \left[\frac{\mathcal{C}_V}{\Lambda^{d_{\mathcal{U}}-1}} (i\varepsilon_{\mu}^* (M_B + M_{K_1}) V_1 (q^2) - i (p + p')_{\mu} (\varepsilon^* \cdot q) \frac{V_2 (q^2)}{M_B + M_{K_1}} \right. \\ &\quad \left. - i q_{\mu} (\varepsilon^* \cdot q) \frac{2M_{K_1}}{q^2} (V_3 (q^2) - V_0 (q^2)) + \frac{\mathcal{C}_A}{\Lambda^{d_{\mathcal{U}}-1}} \left(\frac{2A (q^2)}{M_B + M_{K_1}} \epsilon_{\mu\nu\alpha\beta} \varepsilon^{\nu*} p^{\alpha} p'^{\beta} \right) \right] O_{\mathcal{U}}^{\mu} \end{aligned} \quad (24)$$

and the differential decay rate will be:

$$\begin{aligned} \frac{d\Gamma^{V\mathcal{U}}}{dE_{K_1}} &= \frac{1}{8\pi^2 m_B} \frac{A_{d_{\mathcal{U}}}}{\Lambda^{2d_{\mathcal{U}}-2}} \sqrt{E_{K_1}^2 - M_{K_1}^2} (q^2)^{d_{\mathcal{U}}-2} \\ &\left[8 |\mathcal{C}_A|^2 M_B^2 (E_{K_1}^2 - M_{K_1}^2) \frac{A (q^2)}{(M_B + M_{K_1})^2} \right. \\ &\quad \left. + |\mathcal{C}_V|^2 \frac{1}{M_{K_1}^2 (M_B + M_{K_1})^2 q^2} \right. \\ &\quad \left[(M_B + M_{K_1})^4 (3M_{K_1}^4 + 2M_B^2 M_{K_1}^2 - 6M_B M_{K_1}^2 E_{K_1} + M_B^2 E_{K_1}^2) |V_1 (q^2)|^2 \right. \\ &\quad \left. + 2M_B^4 (E_{K_1}^2 - M_{K_1}^2) |V_2 (q^2)|^2 + 4 (M_B + M_{K_1})^2 \right. \\ &\quad \left. \left. (M_B E_{K_1} - M_{K_1}^2) (M_{K_1}^2 - E_{K_1}^2) M_B^2 (V_1 V_2^* + V_2 V_1^*) \right] \right] \end{aligned} \quad (25)$$

The total decay width can be obtained if we integrate on the energy of the final state meson in the range $M_{K(K_1)} < E_{K(K_1)} < (M_B^2 + M_{K(K_1)}^2) / 2M_B$ for $B \rightarrow K(K_1) + \mathbb{Z}$.

Recently, Grinstein et al. have given comment on the unparticle [26] in which they have mentioned that Mack's unitarity constraint lower bounds on CFT operator dimensions, e.g. $d_{\mathcal{U}} \geq 3$ for primary, gauge invariant, vector unparticle operators. To account for this they have corrected the results in the literature, and modified the propagator of vector and tensor unparticles. The modified vector propagator is

$$\int d^4x e^{iPx} \langle 0 | T (O_{\mathcal{U}}^\mu(x) O_{\mathcal{U}}^\nu(x)) | 0 \rangle = A_{d_{\mathcal{U}}} (-g^{\mu\nu} + a P^\mu P^\nu / P^2) (P^2)^{d_{\mathcal{U}}-2}. \quad (26)$$

Here P is the momentum of the unparticle, $A_{d_{\mathcal{U}}}$ is defined in Eq. (8) and $a \neq 1$ (in contrast to the value $a = 1$ which was considered by Georgi [14]) but is defined as:

$$a = \frac{2(d_{\mathcal{U}} - 2)}{(d_{\mathcal{U}} - 1)}. \quad (27)$$

By incorporating this factor a in the vector unparticle operator the Eqs. (23) and (25) get modification and the modified result of the decay rate for $B \rightarrow K_0^* \mathcal{U}$ is

$$\begin{aligned} \frac{d\Gamma^{V\mathcal{U}}}{dE_{K_0^*}} &= \frac{1}{8\pi^2 m_B} \frac{A_{d_{\mathcal{U}}}}{\Lambda^{2d_{\mathcal{U}}-2}} |C_A|^2 |f_+(q^2)|^2 \left(M_B^2 + M_{K_0^*}^2 - 2M_B E_{K_0^*} \right)^{d_{\mathcal{U}}-2} \sqrt{E_{K_0^*}^2 - M_{K_0^*}^2} \\ &\quad \left[|f_+(q^2)|^2 \left(- \left(M_B^2 + M_{K_0^*}^2 + 2M_B E_{K_0^*} \right) + \frac{a \left(M_B^2 - M_{K_0^*}^2 \right)^2}{\left(M_B^2 + M_{K_0^*}^2 - 2M_B E_{K_0^*} \right)} \right) \right. \\ &\quad + |f_-(q^2)|^2 (a-1) \left(M_B^2 + M_{K_0^*}^2 - 2M_B E_{K_0^*} \right) \\ &\quad \left. + 2(a-1) (f_+(q^2) f_-(q^2)) \left(M_B^2 - M_{K_0^*}^2 \right) \right] \end{aligned} \quad (28)$$

Similarly, for $B \rightarrow K_1 \mathcal{U}$ the result becomes

$$\begin{aligned} \frac{d\Gamma^{V\mathcal{U}}}{dE_{K_1}} &= \frac{1}{8\pi^2 m_B} \frac{A_{d_{\mathcal{U}}}}{\Lambda^{2d_{\mathcal{U}}-2}} \sqrt{E_{K_1}^2 - M_{K_1}^2} (q^2)^{d_{\mathcal{U}}-2} \\ &\quad \left[|\mathcal{M}_{11}|^2 + |\mathcal{M}_{22}|^2 + |\mathcal{M}_{33}|^2 + |\mathcal{M}_{44}|^2 + |\mathcal{M}_{23}|^2 + |\mathcal{M}_{24}|^2 + |\mathcal{M}_{34}|^2 \right] \end{aligned} \quad (29)$$

with

$$\begin{aligned}
|\mathcal{M}_{11}|^2 &= 8 |\mathcal{C}_A|^2 M_B^2 (E_{K_1}^2 - M_{K_1}^2) \frac{A(q^2)}{(M_B + M_{K_1})^2} \\
|\mathcal{M}_{22}|^2 &= |\mathcal{C}_V|^2 \frac{1}{M_{K_1}^2 (M_B + M_{K_1})^2 q^2} \\
&\quad \left[(M_B + M_{K_1})^4 (3M_{K_1}^2 (M_B^2 + M_{K_1}^2 - 2M_B E_{K_1}) - a (M_B^2 M_{K_1}^2 - M_B^2 E_{K_1}^2)) |V_1(q^2)|^2 \right] \\
|\mathcal{M}_{33}|^2 &= |\mathcal{C}_V|^2 \frac{1}{M_{K_1}^2 (M_B + M_{K_1})^2 q^2} \\
&\quad \left[M_B^2 (E_{K_1}^2 - M_{K_1}^2) \left(a (M_B^2 - M_{K_1}^2)^2 + (2M_B E_{K_1})^2 - (M_B^2 + M_{K_1}^2)^2 \right) |V_2(q^2)|^2 \right] \\
|\mathcal{M}_{44}|^2 &= |\mathcal{C}_V|^2 \frac{1}{M_{K_1}^2 (M_B + M_{K_1})^2 q^2} \\
&\quad \left[4M_B^2 (M_B + M_{K_1})^2 (E_{K_1}^2 - M_{K_1}^2) (a - 1) M_{K_1}^2 |V_3(q^2) - V_0(q^2)|^2 \right] \\
|\mathcal{M}_{23}|^2 &= |\mathcal{C}_V|^2 \frac{1}{M_{K_1}^2 (M_B + M_{K_1})^2 q^2} \\
&\quad \left[M_B^2 (M_B + M_{K_1})^2 (E_{K_1}^2 - M_{K_1}^2) (M_B^2 + M_{K_1}^2 - 2M_B E_{K_1} - a (M_B^2 - M_{K_1}^2)) \right. \\
&\quad \left. (V_1(q^2) V_2^*(q^2) + V_2(q^2) V_1^*(q^2)) \right] \\
|\mathcal{M}_{24}|^2 &= |\mathcal{C}_V|^2 \frac{1}{M_{K_1}^2 (M_B + M_{K_1})^2 q^2} \\
&\quad \left[2M_{K_1} (M_B + M_{K_1})^3 ((1 - a) M_B^2 (E_{K_1}^2 - M_{K_1}^2)) (V_1 (V_3 - V_0)^* + (V_3 - V_0) V_1^*) \right] \\
|\mathcal{M}_{34}|^2 &= |\mathcal{C}_V|^2 \frac{1}{M_{K_1}^2 (M_B + M_{K_1})^2 q^2} \left[2M_{K_1} (M_B + M_{K_1}) (M_B^2 - M_{K_1}^2) \right. \\
&\quad \left. M_B^2 (E_{K_1}^2 - M_{K_1}^2) (a - 1) (V_2 (V_3 - V_0)^* + (V_3 - V_0) V_2^*) \right] \tag{30}
\end{aligned}$$

One can easily see that Eqs. (28) and (29) reduces to the Eqs. (23) and (25) respectively, if one puts $a = 1$.

IV. RESULTS AND DISCUSSIONS

In this section we present our numerical study for the $B \rightarrow K_0^*(K_1) + \cancel{E}$ where we try to distinguish the unparticle physics effects from that of the SM. In Standard Model \cancel{E} which is the missing energy is attributed to the neutrinos where as in the case under consideration, this is attached to the unparticle. Therefore the total decay rate can be written as

$$\Gamma = \Gamma^{SM} + \Gamma^{\mathcal{U}}. \tag{31}$$

Here Γ^{SM} is the Standard Model contribution ($B \rightarrow K_0^* (K_1) \nu \bar{\nu}$) where as $\Gamma^{\mathcal{U}}$ is from the unparticle ($B \rightarrow K_0^* (K_1) \mathcal{U}$) to the decay $B \rightarrow K_0^* (K_1) + \cancel{E}$. In ref. [25] it is pointed out that the SM process $B \rightarrow K (K^*) \nu \bar{\nu}$ provides a unique energy distribution spectrum of final state hadrons and present experimental limits on the branching ratio of these processes are about an order of magnitude below the respective SM expectation values. They have used experimental upper limit on the branching ratio of $B \rightarrow K (K^*) \nu \bar{\nu}$ decay to estimate the constraints on the unparticle properties.

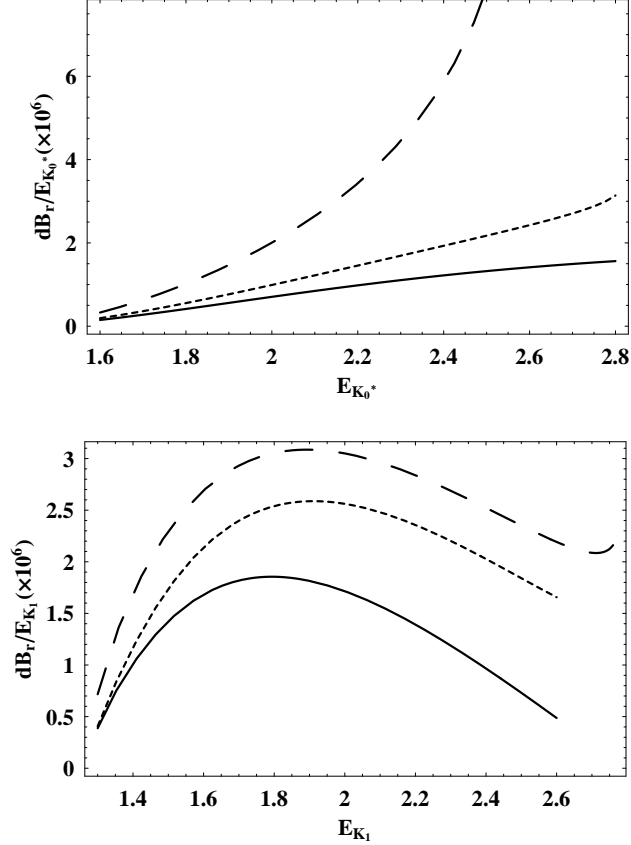


FIG. 1: The differential branching ratio for $B \rightarrow K_0^* (K_1) + \cancel{E}$ as a function of hadronic energy $E_{K_0^*} (E_{K_1})$ is plotted. Top panel is for $B \rightarrow K_0^* + \cancel{E}$ and bottom is for $B \rightarrow K_1 + \cancel{E}$. The other parameters are $d_{\mathcal{U}} = 1.9$, $\Lambda_{\mathcal{U}} = 1000$ GeV, $C_P = C_S = 2 \times 10^{-3}$ and $C_V = C_A = 10^{-5}$. Solid line is for SM, dashed line is for scalar operator and long-dashed line is for the vector operator.

In case of $B \rightarrow K_0^* (K_1) \nu \bar{\nu}$ there is no experimental limit on the branching ratio of these decays, but these will be expected to be measured at future SuperB factories where they analyze the spectra of final state hadron by imposing a cut of on the high momentum of hadron to reduce the background. To calculate the numerical value of the branching ratio for $B \rightarrow K_0^* (K_1) \nu \bar{\nu}$ in SM we have to integrate Eqs. (10) and (15) on the energy of the final state hadron. Thus after

integration, the values of the branching ratios in SM are:

$$\begin{aligned}\mathcal{Br}(B \rightarrow K_0^* \nu \bar{\nu}) &= 1.12 \times 10^{-6} \\ \mathcal{Br}(B \rightarrow K_1 \nu \bar{\nu}) &= 1.77 \times 10^{-6}\end{aligned}\tag{32}$$

With these values at hand, we have plotted the differential decay with for $B \rightarrow K_0^*(K_1) + \cancel{E}$ as a function of the energy of the final state hadron $E_{K_0^*}(E_{K_1})$ and by fixing the parameters of unparticle from ref. [25] in Fig.1. One can easily see from the figure that the signature of unparticle operators are very distinctive from the SM for the final state hadron's energy. Just like $B \rightarrow K(K^*) + \cancel{E}$ the distribution of unparticle contribution is quite different when we include a vector operator ($a = 1$) for the highly energetic final state hadron. For the other values of a we will discuss this issue separately. Thus the Super B-factories will be able to clearly distinguish the presence of unparticle by observing the spectrum of final state hadrons in $B \rightarrow K_0^*(K_1) + \cancel{E}$ in complement to $B \rightarrow K(K^*) + \cancel{E}$.

In Fig. 2 and Fig. 3 we have shown the sensitivity of the branching ratio on the scaling dimension $d_{\mathcal{U}}$ for different values of the cut-off scale $\Lambda_{\mathcal{U}}$ by using the same values of C_S , C_P , C_V and C_A as we have used for Fig. 1. We can see from this figure that the branching ratio is very sensitive to the variable $d_{\mathcal{U}}$ and $\Lambda_{\mathcal{U}}$. The constraints on the vector operator are more stronger than the scalar operators and constraints for $B \rightarrow K_0^* + \cancel{E}$ are better than the $B \rightarrow K_1 + \cancel{E}$ decays.

After showing the dependence of branching ratio on $d_{\mathcal{U}}$ and $\Lambda_{\mathcal{U}}$ what we have shown in Fig. 4 is the sensitivity of the branching ratio of $B \rightarrow K_0^* + \cancel{E}$ with the effective coupling constants of scalar and vector unparticle operators. One can see that $B \rightarrow K_0^* +$ scalar unparticle operator shall constrain the parameter C_P and $B \rightarrow K_0^* +$ vector unparticle operator shall constrain the parameter C_A . Thus observing this decay we can get some useful constraint on C_P and C_A which provides us the signature about the unparticle physics. Similarly, we have shown the dependence of the branching ratio of $B \rightarrow K_1 + \cancel{E}$ on the effective coupling constants in Fig. 5. It is shown that if we consider the scalar operator then only dependence is on C_S , whereas if we consider the vector operators then the decay rate depends both on C_V and C_A .

As we have already mentioned that, Grinstein et al. have recently given their comment on the unparticle [26] mentioning that one regards Mack's unitarity constraint lower bounds on CFT operator dimensions, e.g., $d_{\mathcal{U}} \geq 3$ for primary, gauge invariant, vector unparticle operators. To account for this they have corrected the results in the literature, and modified the propagator of vector and tensor unparticles. The modified expressions of decay rate for the processes under consideration are given in Eq. (28) and Eq. (29). To incorporate this modification in vector

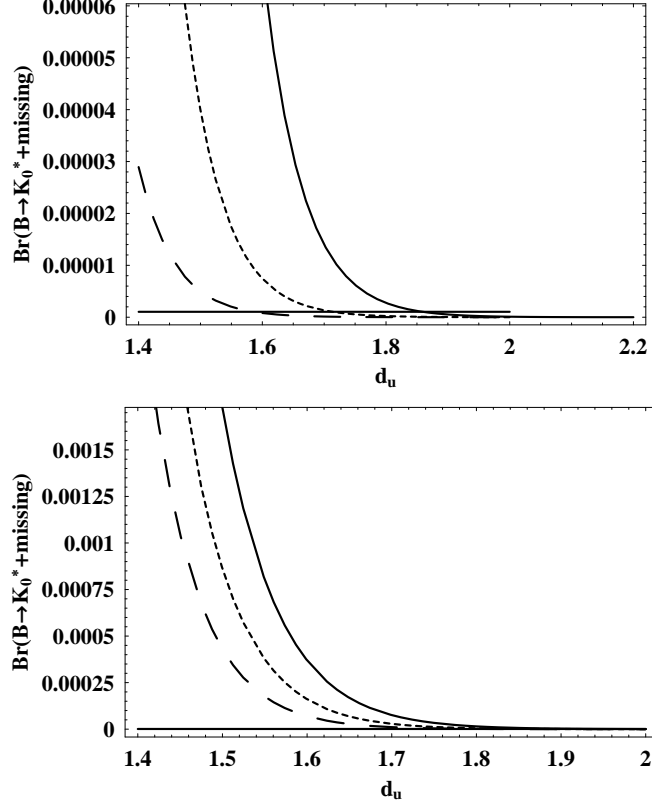


FIG. 2: The branching ratio for $B \rightarrow K_0^* + \cancel{E}$ as a function of $d_{\mathcal{U}}$ for various values of $\Lambda_{\mathcal{U}}$. Top panel is for the scalar operator and bottom is for the vector operator. The values of coupling constants is same as taken for Fig. 1. Solid line is for $\Lambda_{\mathcal{U}} = 1000$ GeV, dashed line is for $\Lambda_{\mathcal{U}} = 2000$ GeV and long-dashed line is for $\Lambda_{\mathcal{U}} = 5000$ GeV. The horizontal solid line is the SM result.

unparticle operator, what we have shown in Fig. 6 is the fractional error

$$\Delta \equiv \frac{\left(\frac{1}{\Gamma} \frac{d\Gamma}{dE_{K_0^*(K_1)}} \right)_{a=1} - \left(\frac{1}{\Gamma} \frac{d\Gamma}{dE_{K_0^*(K_1)}} \right)_a}{\left(\frac{1}{\Gamma} \frac{d\Gamma}{dE_{K_0^*(K_1)}} \right)_{a=1}} \quad (33)$$

where the difference is between the spectrum of $B \rightarrow K_0^*(K_1)\mathcal{U}$ using vector unparticle operator with $a = 1$ and $a = 2(d_{\mathcal{U}} - 2)/(d_{\mathcal{U}} - 1)$ with $3 < d_{\mathcal{U}} < 3.9$. It is clear from the graph that if we increase the unparticle scaling dimensions $d_{\mathcal{U}}$ the contribution of vector unparticle operator to the decay rate decreases significantly because of the increase in the inverse powers of cutoff scale $\Lambda_{\mathcal{U}}$ (see Eqs. (28) and (29)).

Just to conclude: The study of these p-wave decays of B mesons will not only provide us information about SM but it also indicate the physics beyond it and in future, when enough data

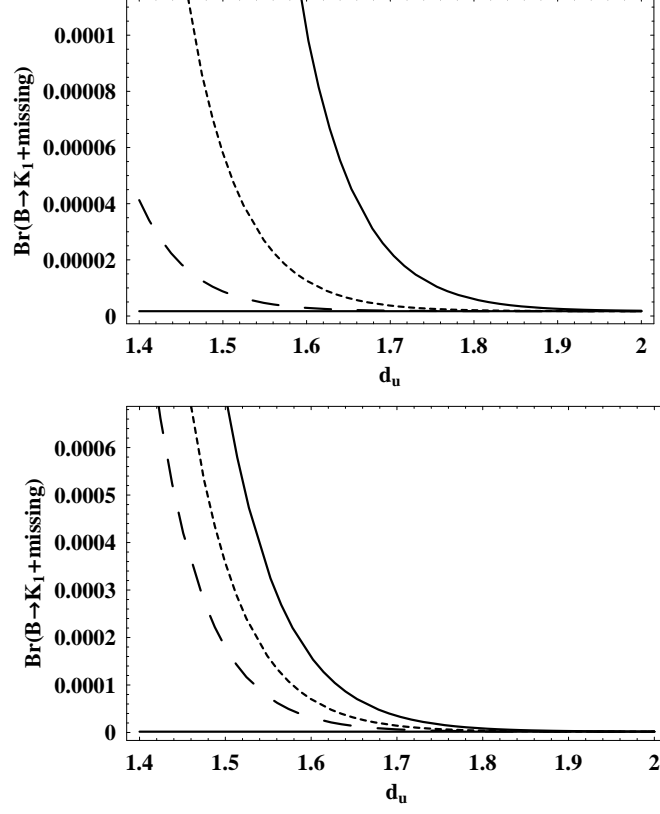


FIG. 3: The branching ratio for $B \rightarrow K_1 + \cancel{E}$ as a function of d_u for various values of Λ_U . Top panel is for the scalar operator and bottom is for the vector operator. The values for coupling constants is same as taken for Fig. 1. Solid line is for $\Lambda_U = 1000$ GeV, dashed line is for $\Lambda_U = 2000$ GeV and long-dashed line is for $\Lambda_U = 5000$ GeV. The horizontal solid line is the SM result.

is expected from the Super B factories, we believe that these decays will take us step forward to the study of unparticle as a source of missing energy in flavor physics.

Acknowledgements:

This work is partly supported by National Science Foundation of China under the Grant Numbers 10735080 and 10625525. The authors would like to thank W. Wang and Yu-Ming for useful discussions.

-
- [1] B. Aubert et al. (BABAR Collaboration), Phys. Rev. Lett. **93**, 081802 (2004)
 - [2] M. I. Iwasaki et al. (BELLE Collaboration), Phys. Rev. **D72**, 092005 (2005).
 - [3] K. Abe et al. (BELLE Collaboration), Phys. Rev. Lett. **88**, 021801 (2002).
 - [4] B. Aubert et al. (BABAR Collaboration), Phys. Rev. Lett. **91**, 221802 (2003).

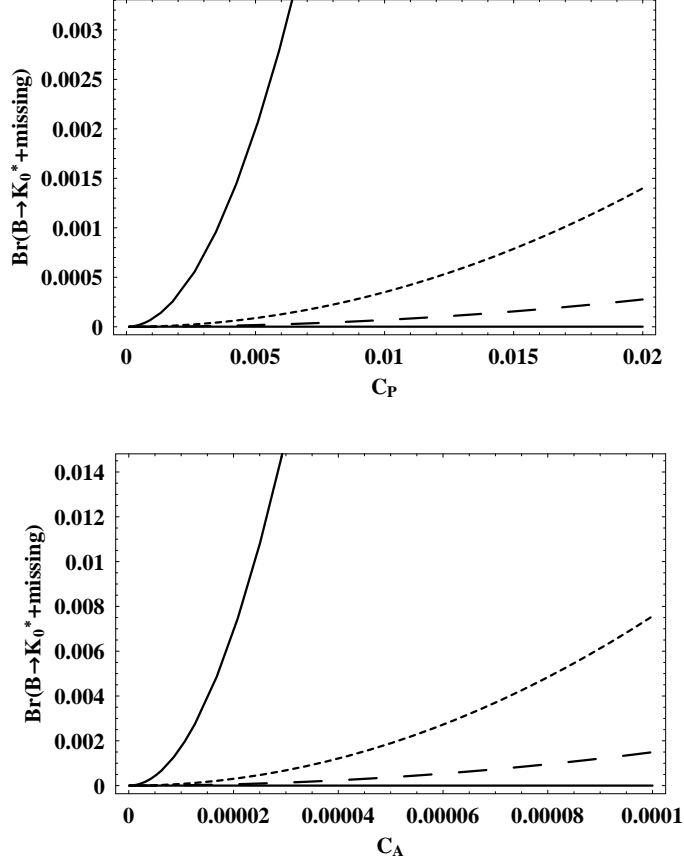


FIG. 4: The branching ratio for $B \rightarrow K_0^* + \cancel{E}$ as a function of C_P (top panel) and C_A (bottom panel). The cut off scale has been taken to be $\Lambda_U = 1000$ GeV. Solid line is for $d_U = 1.5$, dashed line is for $d_U = 1.7$ and long-dashed line is for $d_U = 1.9$. The horizontal solid line is the SM result.

- [5] A. Ishikawa et al. (BELLE Collaboration), Phys. Rev. Lett. **91**, 261601 (2003).
- [6] P. Colangelo, F. De Fazio, P. Santorelli, and E. Scrimieri, Phys. Rev. **D53**, 3672 (1996); **57**, 3186(E) (1998); A. Ali, P. Ball, L. T. Handoko, and G. Hiller, Phys. Rev. **D61**, 074024 (2000); A. Ali, E. Lunghi, C. Greub, and G. Hiller, Phys. Rev. **D66**, 034002 (2002).
- [7] T. M. Aliev, H. Koru, A. O' zpineci, and M. Savc , Phys. Lett. **B400**, 194 (1997); T. M. Aliev, A. O' zpineci, and M. Savc , Phys. Rev. **D56**, 4260 (1997); D. Melikhov, N. Nikitin, and S. Simula, Phys. Rev. **D57**, 6814 (1998).
- [8] G. Burdman, Phys. Rev. **D52**, 6400 (1995); J. L. Hewett and J. D. Walls, Phys. Rev. **D55**, 5549 (1997); C. H. Chen and C. Q. Geng, Phys. Rev. **D63**, 114025 (2001).
- [9] G. Buchalla, G. Hiller and G. Isidori, Phys. Rev. **D63**, 014015 (2001).
- [10] Kai-Feng Chen [Belle Collaboration] Talk given at FPCP07, Slovenia, May 12-16, 2007.
- [11] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. **94**, 101801 (2005).
- [12] C. S. Kim, Y. G. Kim adn T. Morozumi, Phys. Rev. **D60**, 094007 (1999) [arXiv: hep-ph/9905528].

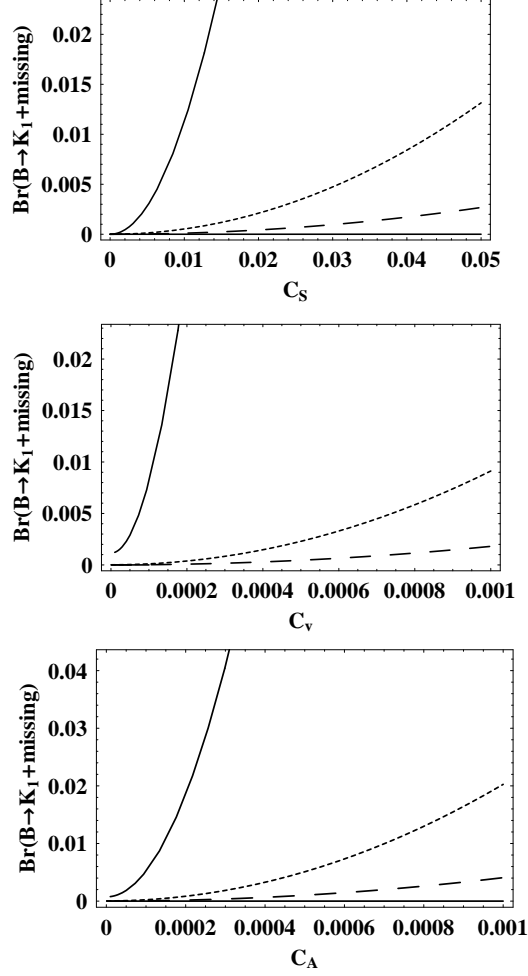


FIG. 5: The branching ratio for $B \rightarrow K_1 + \cancel{E}$ as a function of C_S (top panel), C_A (middle panel) and C_V (bottom panel). The cut off scale has been taken to be $\Lambda_U = 1000$ GeV. Solid line is for $d_U = 1.5$, dashed line is for $d_U = 1.7$ and long-dashed line is for $d_U = 1.9$. The horizontal solid line is the SM result.

- [13] N. Mahajan, Phys. Rev. **D68**, 034012 (2003).
- [14] H. Georgi, Phys. Rev. Lett. **98**, 221601(2007) [arXiv: hep-ph/0703260].
- [15] T. Banks and A. Zaks, Nucl. Phys. **B196**, 189 (1982).
- [16] H. Georgi, Phys. Lett. **B650**, 275-278 (2007) [arXiv: hep-ph/07042457].
- [17] K. Cheung, W.-Y. Keung and T.-C. Yuan, Phys. Rev. Lett. **99**, 051803 (2007): arXiv: hep-ph/0704.2588.
- [18] K. Cheung, W.-Y. Keung and T.-C. Yuan, Phys. Rev. **D76**, 055003 (2007); arXiv: hep-ph/0706.3155.
- [19] P.J. Fox, A. Rajaraman and Y. Shirman, arXiv:0705.3092; N. Greiner, arXiv:0705.3518; S.-L. Chen and X.-G. He, arXiv:0705.3946; G.-J. Ding and M.-L. Yan, arXiv:0705.0794.
- [20] T. Kikuchi and N. Okada, arXiv: hep-ph/0707.0893; T. Kikuchi and N. Okada, arXiv: hep-ph/0801.0018 .

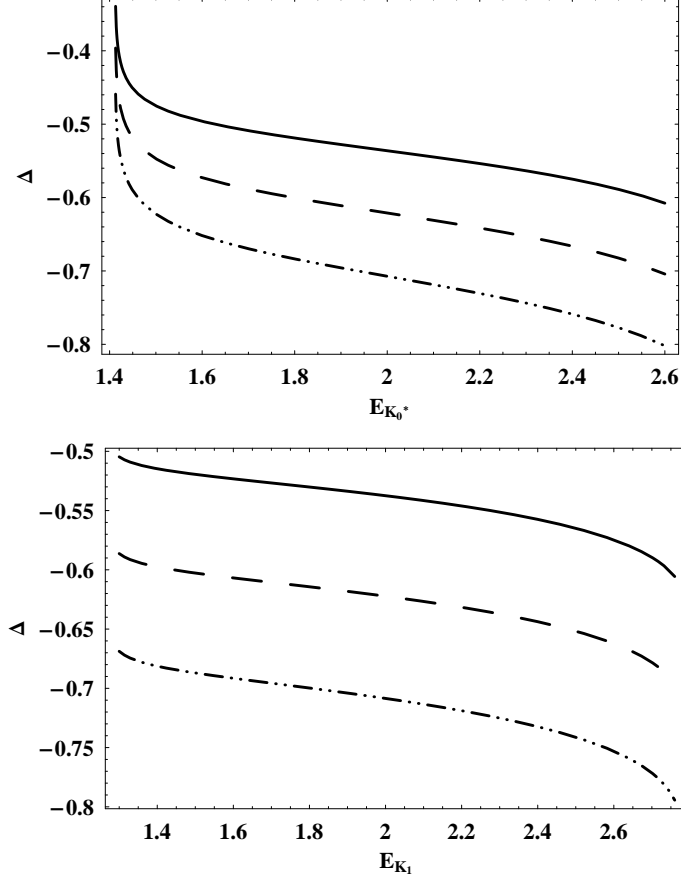


FIG. 6: Fractional error Δ in the spectrum for the decay $B \rightarrow K_0^*(K_1) + \text{vector unparticle operator}$ as a function of energy of final state hadron. Top panel shows the value for $B \rightarrow K_0^*$ and the bottom panel is for $B \rightarrow K_1$. The values for the coupling constants and cutoff scale is same as taken for Fig. 1. Solid line is for $d_U = 3.2$, dashed line is for $d_U = 3.4$ and dashed-double dotted is for $d_U = 3.6$.

- [21] M. Luo and G. Zhu, arXiv:0704.3532; C.-H. Chen and C.-Q. Geng, arXiv:0705.0689; Y. Liao, arXiv: hep-ph/0705.0837; X.-Q. Li and Z.-T. Wei, arXiv:hep-ph/0705.1821.
- [22] D. Choudhury, D.K. Ghosh and Mamta, arXiv: hep-ph/0705.3637. T.M. Aliev, A.S. Cornell and N. Gaur, Phys. Lett. **B657**, 77 (2007) :arXiv: hep-ph/0705.1326. C.-D. Lu, W. Wang and Y.-M. Wang, Phys. Rev. **D76**, 077701 (2007): arXiv: hep-ph/0705.2909.
- [23] A. Lenz, Phys. Rev. **D76**, 065006 (2007): arXiv: hep-ph/0707.1535.
- [24] H. Davoudiasl, arXiv: hep-ph/0705.3636; T. Kikuchi and N. Okada, arXiv: hep-ph/0711.1506.
- [25] T. M. Aliev, A. S. Cornell and N. Gaur, JHEP **07**, 072 (2007) [arXiv: hep-ph/07054542]
- [26] B. Grinstein, K. Intriligator and I. Z. Rothstein, arXiv: hep-ph/08011140.
- [27] Chuan-Hung Chen, Chao-Qiang Geng, Chong-Chung Lih and Chun-Chu Kiu, arXiv: hep-ph/0703106.
- [28] M. Ali Paracha, Ishtiaq Ahmed, M. Jamil Aslam, Eur. Phys. J. **C52**, 967-973 (2007).